Digital Image Processing

Image Compression

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Preview

- Methods of compressing data prior to storage and / or transmission are of significant practical and commercial
- Image compression addresses the problem of reducing the amount of data required to a digital image .
- The underlying basis of the reduction process is the removal of **redundant** data .

Fundamentals

- The data compression refers to the process of reducing the amount of data required to represent a given quantity of information .

- The difference of data and information .

- Data are the means by which information is conveyed نقل.

- Data redundancy is a central issue in digital image compression

Fundamentals

- The relative data redundancy R_D :

$$R_D = 1 - \frac{1}{C_R}$$

Where C_R is compression ratio .

$$C_R = \frac{n_1}{n_2}$$

n1 , n2 donate the number of information - carrying units in two data set that represent the same information .

n1=n2	CR =1	RD =0	n1 contains no redundant data
n2 << n1	CR → ∞	RD→ 1	significant compress & High redundant data
n2 >> n1	CR →0	RD→ -∞	n2 contains much more data than n1 undesirable case

CR=10 every 10 information in n1 represented by 1 bit in n2 , n1 has 90% redundancy

3 basic Data redundancies

- Coding Redundancy .
- Spatial and Temporal Redundancy.
- i.e. Video sequence (Correlated pixels are not repeated.)
- Irrelevant Information.
- Information that ignored by human visual system

Coding Redundancy

- Lets assume , that a discrete random variable r_K in the interval [0 , 1] represents the gray levels of an image and each r_K occurs with probability

$$P_r(r_K)$$

$$P_r(r_K) = \frac{n_K}{n} \qquad k=0,1,2,\ldots\ldots L-1$$

Where L is the number gray levels ,

n_K is the number of times that the K^{th} gray level appears in image .

n is the total number of pixel in the image .

Coding Redundancy

The average length of the code words assigned to the various gray level values

$$L_{avg} = \sum_{K=0}^{L-1} l(r_K)\, p_r(r_K)$$

where $l(r_k)$ *no.\of bits used to represent each gray*

$p_r(r_k)$ *probability that gray level occurs*

Example

r_k	$p_r(r_k)$	**Code 1**	$l_1(r_k)$	**Code 2**	$l_2(r_k)$
$r_0 = 0$	0.19	000	3	11	2
$r_1 = 1/7$	0.25	001	3	01	2
$r_2 = 2/7$	0.21	010	3	10	2
$r_3 = 3/7$	0.16	011	3	001	3
$r_4 = 4/7$	0.08	100	3	0001	4
$r_5 = 5/7$	0.06	101	3	00001	5
$r_6 = 6/7$	0.03	110	3	000001	6
$r_7 = 1$	0.02	111	3	000000	6

TABLE 8.1 Example of variable-length coding.

$$L_{avg} = \sum l(r_K)\, p_r(r_K)$$

$$= 2(0.19) + 2(0.25) + 2(0.21) + 3(0.16) + 4(0.08)$$

$$+ 5(0.06) + 6(0.03) + 6(0.02)$$

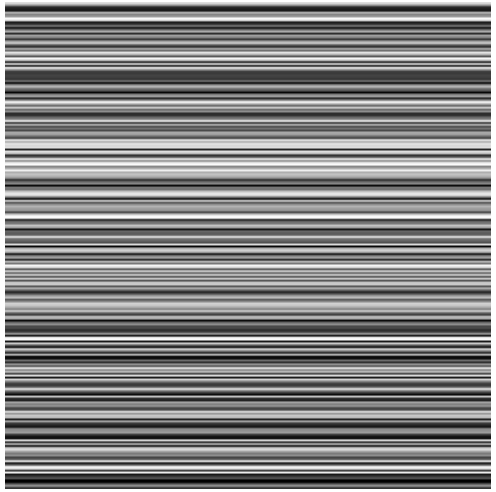
$$= 2.7 bits.$$

Example

- The resulting compression ratio C_R is 3/2.7 or 1.11 .
- Thus approximately 10% of the data resulting from the use of code 1 is redundant .
- The exact level of redundancy can be determine from

$$R_D = 1 - \frac{1}{1.11} = 0.099$$

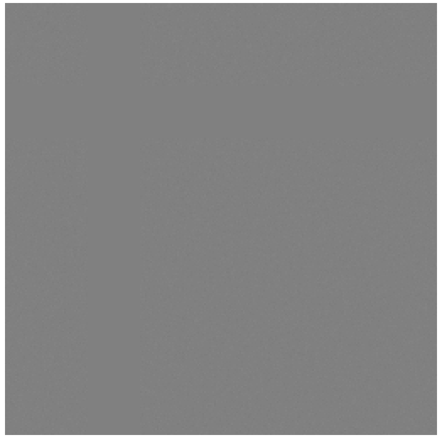
Spatialand Temporal redundancy



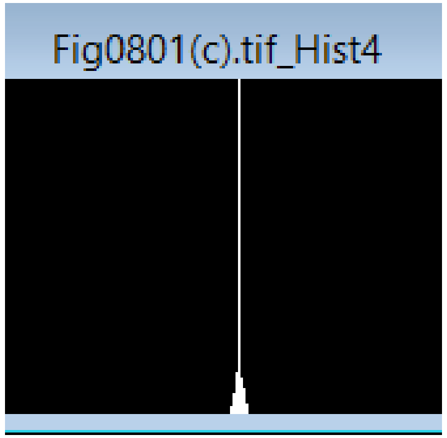
- Each line has the same intensity
- All 256intensity are of equal probability.
- Pixels intensity are independent of each other
- Pixels are correlated vertically
- Pixels intensity can be predicted from their Neighbor intensities, so the information carried   by one pixel is small.

Histogram

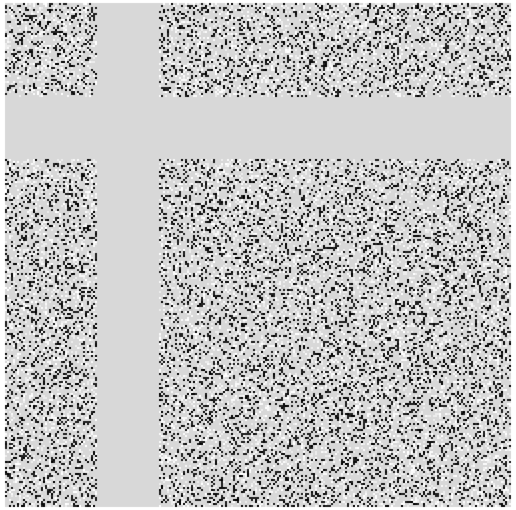
Irrelevant Information



- Information that ignored by HVS are obvious candidates for omission.
- Original size is 256X256X8
- All are seems to be of the same color
- Compression =256X256X8/8 = 65536:1



Irrelevant Information



- This type of redundancy is different from the other 2 types
- Its elimination is possible because the information itself is not essential for HVS.
- Its removal referred to Quantization
- This means mapping of a broad range of intensity into limited range
- Quantization is irreversible operation.

How do we measure information?

- What is the information content of a message/image?
- What is the minimum amount of data that is sufficient to describe completely an image without loss of information?

Modeling Information

Information generation is assumed to be a probabilistic process.

Idea: associate information with probability!

A random event E with probability $P(E)$ contains:

$$I(E) = log(\frac{1}{P(E)}) = -log(P(E)) \text{ units of information}$$

Note: I(E)=0 when P(E)=1 The event always occurs

How much information does a pixel contain?

Suppose that gray level values are generated by a random variable, then r_k contains:

$$I(r_k) = -\,log(P(r_k))$$ units of information!

How much information does an image contain?

Average information content of an image:

[redacted]

using $I(r_k) = -\log(P(r_k))$

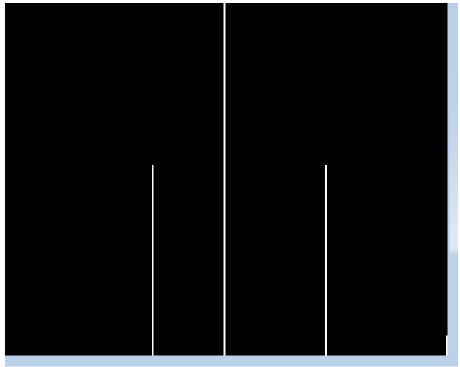
Entropy $H = -\sum_{k=0}^{L-1} P(r_k)\log(P(r_k))$ units/pixel

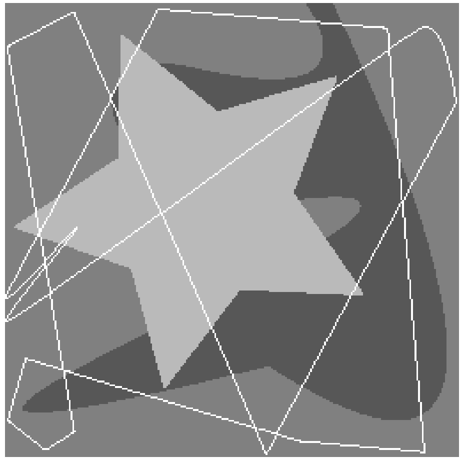
It is not possible to code an image with fewer than H bits/pixel

(assumes statistically independent random events)

Example:

- $H = -[.25 \log_2 0.25 + .47 \log_2 0.47 +$
- $.25 \log_2 0.25 + .03 \log_2 0.03]$
- $= [-0.25(-2) + .47(-1.09) + .25(-2) + .03(-5.06)]$
- $= 1.6614$ bits/pixel





- What about H for the second type of redundancy?

Fidelity Criteria

- Objective fidelity criterion

Loss of information - compress - decompress .

- Subjective fidelity criteria

Quality of image .

Fidelity criteria

- Irrelevant information represents a loss, so we need a mean of quantifying the nature of loss
- When the level of information loss can be expressed as a function of the original or input image and the compressed and decompressed output image , it is based on an objective fidelity criterion .
- Example : the error at any x,y

$$e(x,y) = \underbrace{\hat{f}(x,y)}_{\text{Approximate}} - \underbrace{f(x,y)}_{\text{Original}}$$

- The total error

$$\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\hat{f}(x,y) - f(x,y) \right]$$

- The square root

$$e_{rms} = \left[\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\hat{f}(x,y) - f(x,y) \right]^2 \right]^{1/2}$$

- The mean square signal - to - noise

$$SNR_{rms} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \hat{f}(x,y)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\hat{f}(x,y) - f(x,y) \right]^2}$$

Fidelity Criteria(Subjective)

TABLE 8.3
Rating scale of the Television Allocations Study Organization. (Frendendall and Behrend.)

Value	Rating	Description
1	Excellent	An image of extremely high quality, as good as you could desire.
2	Fine	An image of high quality, providing enjoyable viewing. Interference is not objectionable.
3	Passable	An image of acceptable quality. Interference is not objectionable.
4	Marginal	An image of poor quality; you wish you could improve it. Interference is somewhat objectionable.
5	Inferior	A very poor image, but you could watch it. Objectionable interference is definitely present.
6	Unusable	An image so bad that you could not watch it.

Huffman Coding (coding redundancy)

- A variable-length coding technique.
- Optimal code (i.e., minimizes the number of code symbols per source symbol).

- Assumption: symbols are encoded one at a time!

Huffman Coding (cont'd)

- *<u>Forward Pass</u>*

  1. Sort probabilities per symbol
  2. Combine the lowest two probabilities
  3. Repeat *Step2* until only two probabilities remain.

Original source		Source reduction			
Symbol	Probability	1	2	3	4
a_2	0.4	0.4	0.4	0.4	0.6
a_6	0.3	0.3	0.3	0.3	0.4
a_1	0.1	0.1	0.2	0.3	
a_4	0.1	0.1	0.1		
a_3	0.06	0.1			
a_5	0.04				

Huffman Coding (cont'd)

- ***Backward Pass***

  Assign code symbols going backwards

Original source			Source reduction							
Sym.	Prob.	Code	1		2		3		4	
a_2	0.4	1	0.4	1	0.4	1	0.4	1	0.6	0
a_6	0.3	00	0.3	00	0.3	00	0.3	00	0.4	1
a_1	0.1	011	0.1	011	0.2	010	0.3	01		
a_4	0.1	0100	0.1	0100	0.1	011				
a_3	0.06	01010	0.1	0101						
a_5	0.04	01011								

Huffman Coding (cont'd)

- L_{avg} using Huffman coding:

$$L_{avg} = E(l(a_k)) = \sum_{k=1}^{6} l(a_k)P(a_k) =$$

- L_{avg} assuming binary codes:

6 symbols, we need a 3-bit code

$$(a_1\text{: } 000,\ a_2\text{: } 001,\ a_3\text{: } 010,\ a_4\text{: } 011,\ a_5\text{: } 100,\ a_6\text{: } 101)$$

$$L_{avg} = \sum_{k=1}^{6} l(a_k)P(a_k) = \sum_{k=1}^{6} 3P(a_k) = 3\sum_{k=1}^{6} P(a_k) = 3 \text{ bits/symbol}$$

Huffman Coding/Decoding

- After the code has been created, *coding/decoding* can be implemented using a look-up table.
- Note that decoding is done unambiguously.

$$\underbrace{0\ 1\ 0\ 1\ 0}_{a_3}\ \underbrace{0\ 1\ 1}_{a_1}\ \underbrace{1}_{a_2}\ \underbrace{1}_{a_2}\ \underbrace{0\ 0}_{a_6}$$

Original source		
Sym.	Prob.	Code
a_2	0.4	1
a_6	0.3	00
a_1	0.1	011
a_4	0.1	0100
a_3	0.06	01010
a_5	0.04	01011

Arithmetic (or Range) Coding
(coding redundancy)

- Instead of encoding source symbols one at a time, sequences of source symbols are encoded together.
  - There is no one-to-one correspondence between source symbols and code words.
- Slower than Huffman coding but typically achieves better compression.

Arithmetic Coding (cont.)

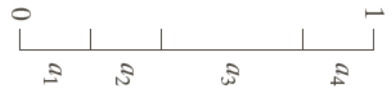
Encode message: $\alpha_1\ \alpha_2\ \alpha_3\ \alpha_3\ \alpha_4$

Source Symbol	Probability
a_1	0.2
a_2	0.2
a_3	0.4
a_4	0.2

1) Start with interval [0, 1)

0 1

2) Subdivide [0, 1) based on the probabilities of α_i



Initial Subinterval
[0.0, 0.2)
[0.2, 0.4)
[0.4, 0.8)
[0.8, 1.0)

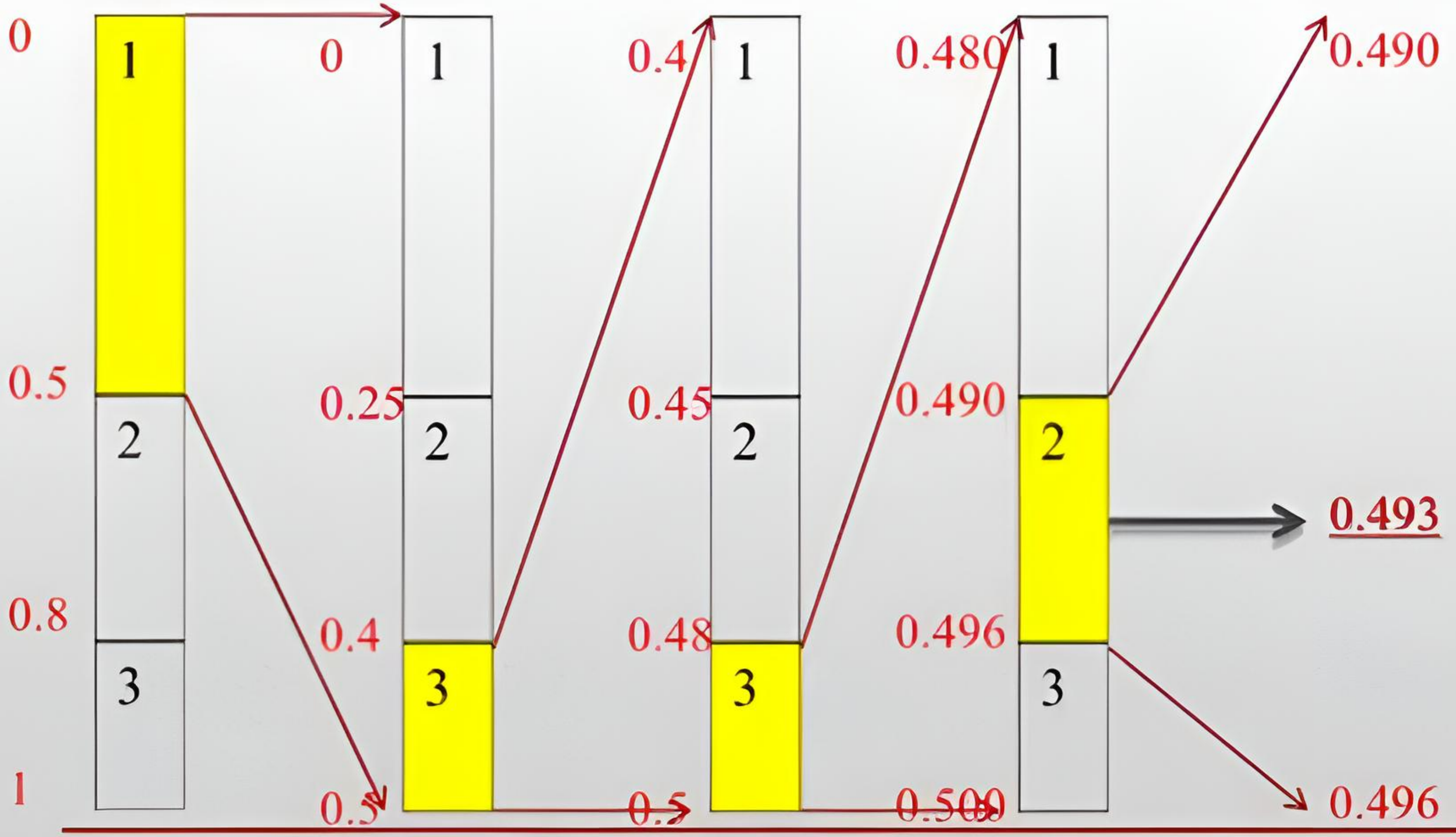
3) Update interval by processing source symbols

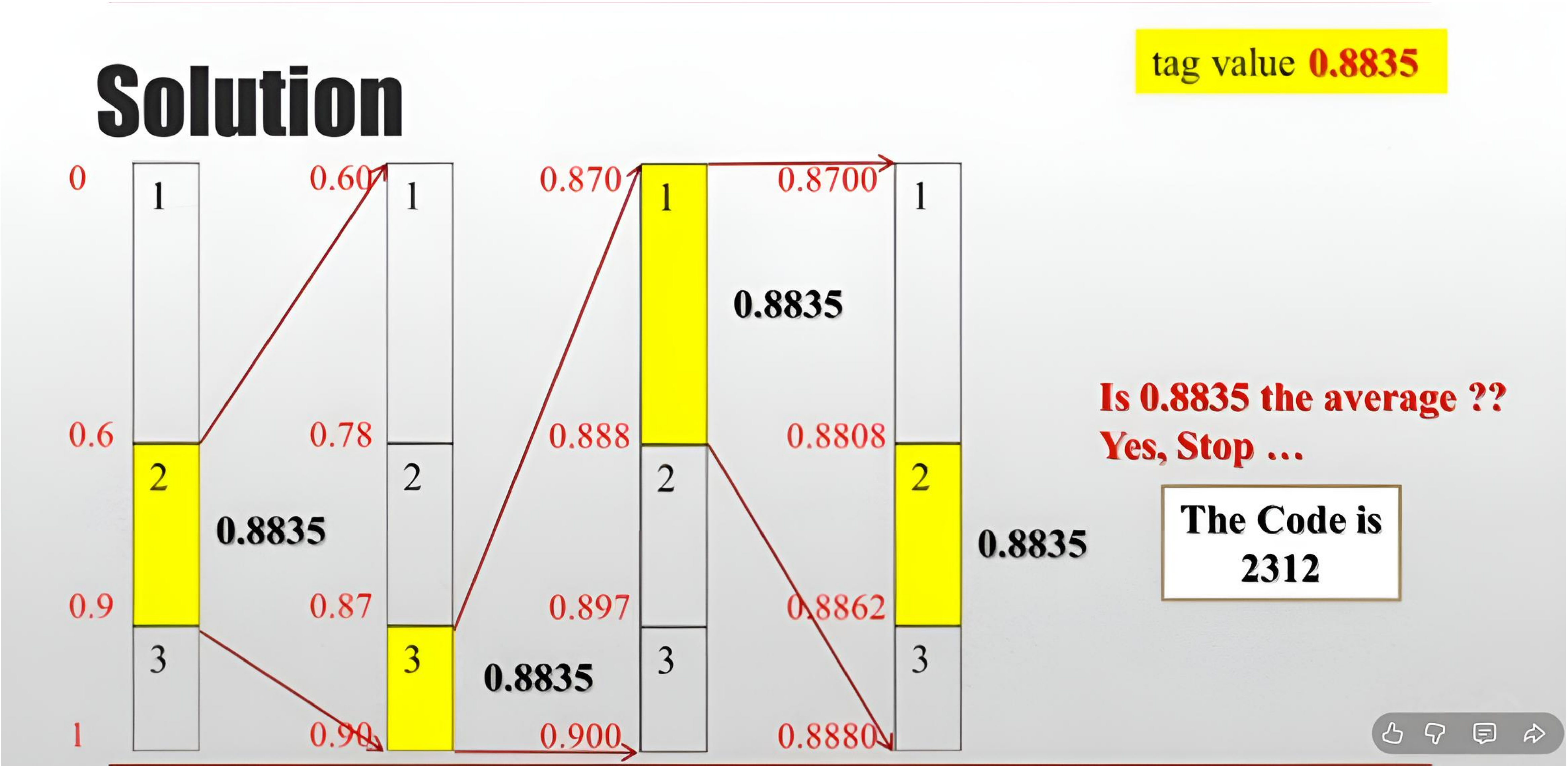
Decoder Example :

- Decipher the tag value **0.8835** for the symbols {1,2,3}
- knowing that :
- $P(1) = 0.6$, $p(2) = 0.3$, $p(3) = 0.1$

Solution

Encode the stream : 1332




Solution
tag value 0.8835
0
0.6
0.9
1
1
2
3
0.8835
0.60
0.78
0.87
0.90
0.8835
0.870
0.888
0.897
0.900
0.8835
0.8700
0.8808
0.8862
0.8880
0.8835
Is 0.8835 the average ??
Yes, Stop …
The Code is
2312